University of Warwick

EC312 International Economics

Supplementary Document for Topic 6

Review the ISLMBP Model

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ABSTRACT

This document aims to solve the ambiguity arisen from the ISLMBP Model lectures, by reviewing the Model and deriving the theorem which clarifies the ambiguity for one particular case. This document also contains an example of the Model with a graph.

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 $[^]a\,\mathrm{I}$ thank Dr Natalie Chen for her wonderful teaching in EC204 and EC312.

1 Introduction

During the learning of Topic 6, confusions arise over some cases of the ISLMBP analyses. Most of the confusions are around how shall the lines shift given some sets of conditions.

One of the examples shown in lecture, i.e. Monetary Expansion under Floating Exchange Rates and Imperfect Capital Mobility, was ambiguous — after monetary expansion, as exchange rate depreciates, IS and BP would shift rightward, but then the eventual change of interest rate comes to be controversial. "Intuitively", monetary expansion never leads the interest rate to go up; however, the IS and BP may shift far right, such that the eventual equilibrium would reflect the interest rate to go up after the monetary expansion.

In the lecture note, Dr Chen took the "intuitive" case, which is that IS and BP shift slightly rightward such that interest rate still goes down after the monetary expansion. Some students, including the author, have a concern over whether the IS and BP could shift far away such that the other possibility occurs, i.e. interest rate goes up.

Therefore, this document shows rigorously that, under the mentioned example, in what cases the interest rate would go down and in what cases the interest rate would go up. An example for interest rate to go up is also attached.

2 Review the Model

Let us review the IS curve first:

$$Y = C(Y) + I(Y, r) + G + CA(Y, e)$$
(2.1)

where	Y	is output,	
	C	is cosumption, depends positively on Y ,	
	Ι	is investment, depends positively on Y and negatively on r ,	
	r	is interest rate, ¹	
	G	is the government spending, set to be exogenous throughout the Model,	
	CA	is the current account, depends negatively on Y and positively ² on e .	

LM curve under Floating Exchange Rate:

$$m = L(Y, r) \tag{2.2}$$

where m is the real money supply, L is a function.

BP curve under Imperfect Capital Mobility:

$$CA(Y,e) + KA(r-r^*) = 0$$
 (2.3)

where KA is the capital account, depends positively on $r - r^*$ r^* is the foreign interest rate.

Same as what we have been doing during the past, we assume the above functions to be linear³. Then write Equation 2.1 to:

$$Y = \alpha_0 - \alpha_1 r + \alpha_2 e \tag{2.4}$$

¹ Real interest rate always equal to nominal interest rate because price is fixed in the ISLMBP Model.

² This is not trivial — it is an important assumption we make. This is an implication from Marshall-Lerner Condition. (Actually, because Marshall-Lerner Condition is an approximation, there are chances where Marshall-Lerner Condition holds but e negatively relates to CA. However, for simplification, let us assume that e positively relates to CA.)

³ With appropriate economic parameters, e.g. C(Y) = p + qY where p > 0 and $q \in (0, 1)$. The details of such process are not shown in this document to reduce the tediousness. Please visit the proofs on the author's website if interested.

Write Equation 2.2 to:

$$Y = \beta_0 + \beta_1 r + \beta_2 m \tag{2.5}$$

Write Equation 2.3 to:

$$Y = \theta_0 + \theta_1 e + \theta_2 (r - r^*)$$
(2.6)

Those parametres have properties:⁴

$$\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2, \theta_0, \theta_1, \theta_2 > 0 \tag{2.7}$$

$$\alpha_2 > \theta_1 \tag{2.8}$$

Having the three equations (Equation 2.4, Equation 2.5, and Equation 2.6), and with given $\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2, \theta_0, \theta_1, \theta_2, r^*, m$, we can solve the three variables Y, r, e.

⁴ Please visit the proofs on the author's website if interested.

3 Main Theorem and an Example

Main Theorem 1. In the ISLMBP model, under the case of Floating Exchange Rates and Imperfect Capital Mobility, Monetary Expansion leads to increase in interest rate if and only if

$$\alpha_1 + \beta_1 - \frac{\alpha_2}{\theta_1}\beta_1 + \frac{\alpha_2}{\theta_1}\theta_2 > 0$$

The proof of Main Theorem 1 can be done by solving the systems of equations as mentioned in section 2, then compute $\frac{\partial r}{\partial m}$ to find the condition. The proof is not shown in this document to reduce the tediousness. Please visit the proofs on the author's website if interested.

To further see the implication of Main Theorem 1, we consider the following example:

Example. Let $\alpha_0 = \alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1$, $\beta_0 = 10$, $\theta_0 = \theta_2 = 2$, $\theta_1 = 0.5$, and $r^* = 2.5$.

Suppose the initial m = 20, and monetary expansion increases m to m = 30.

Under m = 20, we have the system IS_0, LM_0, BP_0 as follows:

$$\begin{cases}
IS_0: \quad Y = 1 - r + e \\
LM_0: \quad Y = 30 + r \\
BP_0: \quad Y = 2 + 0.5e + 2(r - 2.5)
\end{cases}$$
(3.1)

The solution is:

$$\begin{cases} r = 9.25 \\ Y = 39.25 \\ e = 47.5 \end{cases}$$

Under m = 30, we have the system IS_1, LM_1, BP_1 as follows:

$$\begin{cases}
IS_1: \quad Y = 1 - r + e \\
LM_1: \quad Y = 40 + r \\
BP_1: \quad Y = 2 + 0.5e + 2(r - 2.5)
\end{cases}$$
(3.2)

The solution is:

$$\begin{cases} r = 11.75 \\ Y = 51.75 \\ e = 62.5 \end{cases}$$

Clearly, the interest rate increases from 9.25 to 11.75 after the monetary expansion. We can also see such shift in equilibrium from the following graph:



In conclusion, the Main Theorem 1 has clarified the sufficient and necessary condition for the Monetary Expansion leading to a rise in interest rate; additionally, as we can see from the above example that indeed, such possibility exists.