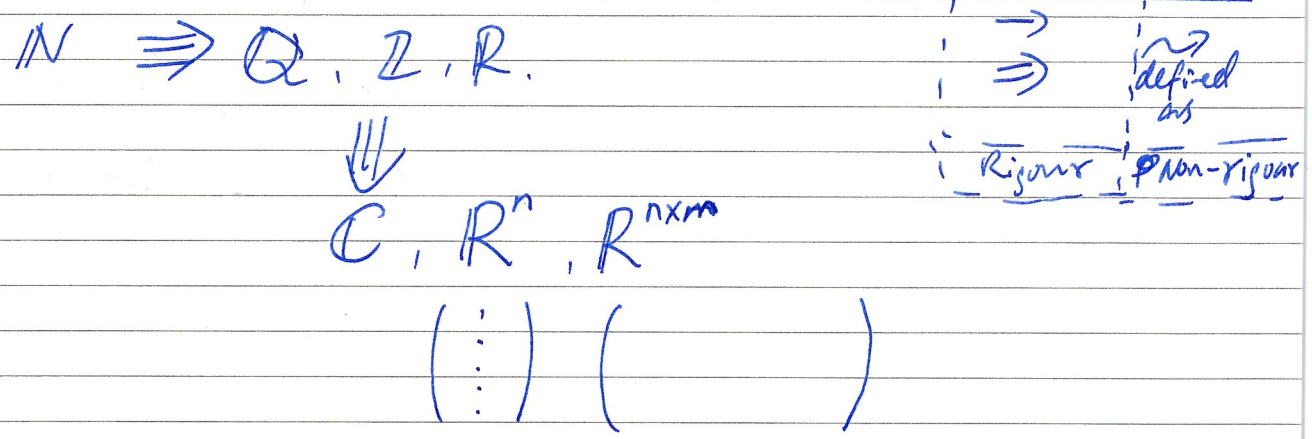


2.1

WMS Mentoring with Henry Webber

27.02.198

- Good spot on those questions!
 - Fundamental
 - Hard
 - Critical for future applied field.
- Abstract Algebra
 - * Field (See page 12/13)
 - * Vector Space (See page 13/14)



P.2

5 ∈ Z ⊂ Q ⊂ R ⊂ C

HW2.2.4

$$A \in \mathbb{R}^{m \times n}$$

r.r. to

$$M \in \mathbb{R}^{m \times n}$$

$$B$$

r.r. to

$$M$$

$$\begin{pmatrix} 1 & 3 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

$$A \xrightarrow{r.o.} \dots \xrightarrow{r.o.} \dots \xrightarrow{r.o.} M$$

$$B \sim \dots \sim \dots \sim M \sim \dots \sim \dots \sim B$$

$$r_1 \mapsto r_1 - r_2 \quad ; \quad r_1 \mapsto r_1 + r_2$$

$$\begin{pmatrix} 3 & 3 \\ 1 & 4 \end{pmatrix} \begin{matrix} \vdots \\ \vdots \end{matrix} \begin{matrix} \nearrow \\ \nearrow \end{matrix}$$

$$\begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}$$

$$A \xrightarrow{r.o.} \dots \sim M \xrightarrow{r.o.} \dots \xrightarrow{r.o.} B$$

(

→

Claim: All row operations (r.o.) are "invertible."

Define: "invertible" is defined as:

if $f: ?$

What is the rigorous way to define a function?

$$f: A \rightarrow B, \quad x \mapsto \sim$$

Row Operation $\rightsquigarrow f: \mathbb{R}^{n \times m} \rightarrow \mathbb{R}^{n \times m}$
 $M \mapsto M'$

\exists $g: \mathbb{R}^{n \times m} \rightarrow \mathbb{R}^{n \times m}$
 $M' \mapsto M$

WLOG, assume r.o. is applying to r_1 .

(R1) $\begin{pmatrix} r_1 \\ \vdots \\ r_n \end{pmatrix} \mapsto \begin{pmatrix} r_1 + ar_i \\ \vdots \\ r_n \end{pmatrix}, \quad a \in \mathbb{R}, \quad 1 \leq i \leq n$

$$\begin{pmatrix} r_1 + ar_i \\ \vdots \\ r_n \end{pmatrix} \stackrel{?}{\mapsto} \begin{pmatrix} r_1 \\ \vdots \\ r_n \end{pmatrix}$$

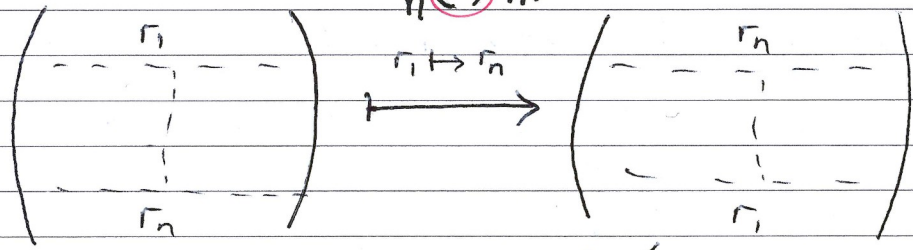
\uparrow
 $r_1 \rightarrow r_1 - ar_i$

$$r_j \mapsto r_j + ar_i$$

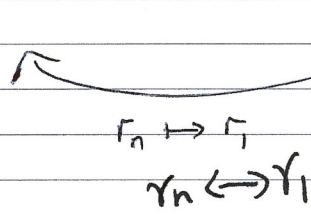
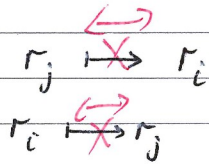
$$r_j \mapsto r_j - ar_i$$

P. 4

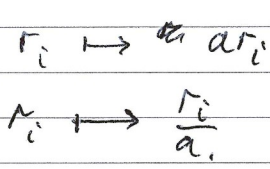
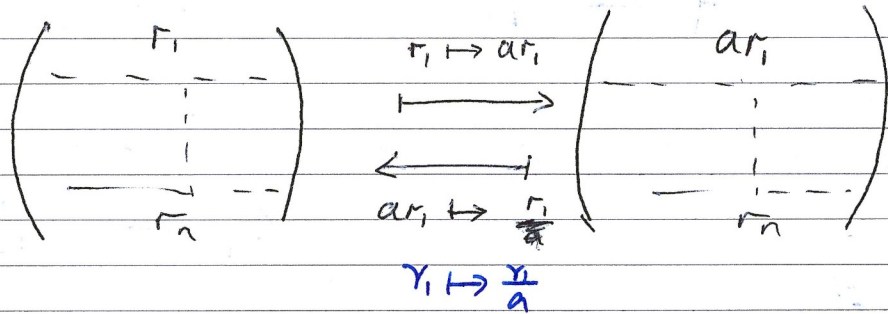
(R2)



$\gamma_1 \mapsto \gamma_n$
 $\gamma_n \mapsto \gamma_1$



(R3)



D