

Does the negative interest rate boost consumption and investment in Japan? — An insight from a Bank-Credit Model

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Resources

Materials including slides for presentations 1 and 2 can be found at: parleyyang.wordpress.com/documents-for-rae-dissertation/

Recall last presentation and outline for this presentation

In presentation 1:

- Motivation.
 - Why Japan? Why Negative Interest Rates? Why Bank-Credit Model?
- Bank-Credit Model.
 - Aim and settings centring at the Commercial Bank.
 - Alex-Jeremy-Bank Story and one of the results.

In this presentation:

- Bank-Credit Model with Macroeconomic elements.
 - General Settings and Individual's Utility Maximisation Problem (UMP).
 - An enhanced version of the Commercial Bank's Balance Sheet accounting.
- Mathematics in solving the problems and Sample Result.
- Further Steps and Extension.
- Conclusion at current stage and Evaluation.

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Introduction to the Macroeconomic elements

Note on the content:

This is a selection of materials from my model. Please contact me if you wish to know more.

Recall aim of the model:

Assess whether the negative interest rate boosts consumption and investment, by considering the commercial bank's credit issuing decisions.

- Assumption on general settings.
 - Consider the economy with two institutions, two types of individuals, and two periods — period 1 and period 2.
 - The first institution is Government-and-Central-Bank, the second institution is a commercial bank (*ComB*).
- Assumptions on credit network, goods market, production, and investment.
- Assumption on types of individuals.
 - Ordinary Farmer (OF): able to deposit and borrow, but unable to invest.
 - Entrepreneurial Farmer (EF): able to deposit, borrow, and invest.

Individual Settings

- Assumption on individual's utility functions.

$$u_j : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}, \quad C_{t;j} \mapsto u_j(C_{t;j}) \quad (1)$$

$$\text{such that } u_j \in C^2(\mathbb{R}_{>0}, \mathbb{R}) \quad (2)$$

$$\forall x \in \mathbb{R}_{>0}, \quad u_j'(x) \geq 0 \quad \text{and} \quad u_j''(x) \leq 0 \quad (3)$$

- Assumption on weather-associated risk.

→ Let weather at period 2 be a binary outcome for each farmer's land.

→ Write X_j to be the weather on the land of the farmer j at period 2.

→ One outcome is good, write as $X_j = G_j$, with probability $p_j \in (0, 1)$.

→ The other outcome is bad, write as $X_j = NG_j$, with the remaining probability $1 - p_j$.

→ Assume the weather on each land is pairwise independent.

- Assumptions on common knowledge, consumption functions, and production functions.

Individual's UMP and Commercial Bank (General)

- Every individual j faces utility maximisation problem (UMP) at period 1 as

$$\max_{C_{1;j} \geq 0 \text{ and } C_{2;j} \geq 0} \left\{ \mathbb{E} \left[u_j(C_{1;j}) + \beta_j u_j(C_{2;j}) \right] \right\} \quad (4)$$

subject to

$$C_{1;j} P_1 \leq F_{1;j} P_1 + B_j - I_j \quad (5)$$

$$C_{2;j} P_2 \begin{cases} = 0 & \text{if } F_{2;j}(NG_j, I_j) P_2 \leq B_j(i_{\#} + 1) \text{ and } X_j = NG_j \\ \leq F_{2;j}(X_j, I_j) P_2 - B_j(i_{\#} + 1) & \text{otherwise} \end{cases} \quad (6)$$

where

$$i_{\#} = \begin{cases} i_{b;j}, & \text{if } B_j > 0 \\ i_{d;j}, & \text{if } B_j \leq 0 \end{cases} \quad (7)$$

- Underlining implication: $F_{2;j}(G_j, I_j) P_2 \geq B_j(i_{\#} + 1)$
- Key Assumptions to the Commercial Bank: utility function u_{ComB} , legal constraints, and balance sheet.

Commercial Bank (Accounting Principle of Balance Sheet)

Balance Sheet of the commercial bank for this paper

Assets	Liabilities
Cash and cash equivalent (A_0)	Deposits from customers (L_0)
Government Bonds and equivalent (A_1)	Equity (L_1)
Lendings (A_2)	

$\forall X \in \{L_0, L_1, A_0, A_1, A_2\}$, write the value of X at period 1 before the rate cut as $X(0)$, and after the rate cut as $X(1)$. Write the value of X at period 2 as $X(2)$.

$$\forall t \in \{0, 1\}, \quad A_0(2) = (1 - c)A_0(t) + \sum_{j=1}^{m+n} (i_{d,j})B_j \mathbb{1}[B_j \leq 0] \quad (8)$$

$$\forall t \in \{0, 1\}, \quad A_1(2) = (1 + i_{c:1})A_1(t) \quad (9)$$

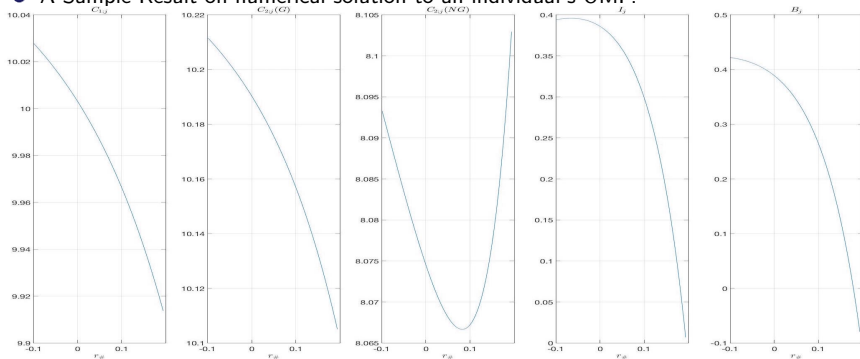
$$\forall t \in \{0, 1\}, \quad A_2(t) = \sum_{j=1}^{m+n} B_j \mathbb{1}[B_j > 0] \quad (10)$$

$$A_2(2) = \sum_{j=1}^{m+n} \left(\left((1 + i_{b,j})B_j(1 - \mathbb{1}[\text{Default}_j]) + F_{2,j}(NG_j, I_j)P_2 \mathbb{1}[\text{Default}_j] \right) \mathbb{1}[B_j > 0] \right) \quad (11)$$

$$\forall t \in \{0, 1, 2\}, \quad L_0(t) = \sum_{j=1}^{m+n} -B_j \mathbb{1}[B_j \leq 0] \quad (12)$$

Mathematics in solving the problems, and a Sample Result

- Theoretical Mathematics: (\mathbb{R}^n, d) metric space and their topological extensions. Convexity, concavity, higher order Fréchet derivatives.
- Practical Trials.
 - Linear utility function & any production function: almost **no interior maximiser**.
 - Any utility function & linear production function: some does not have interior maximisers, some may yield $card(M) = \infty$, i.e. **infinitely many maximisers**.
 - Quadratic utility function & linear production function: **some maximisers are explicit**.
 - Log / power utility function & log / power production function: **nearly all have unique maximisers**, but some maximisers hardly have explicit solutions within one page.
- A Sample Result on numerical solution to an individual's UMP.



Further Steps and Extension

- Further steps. (Ordered in a decreasing priority. Some less prioritised items may be done in my postgraduate research.)
 - **Dynamic**: generalise the time periods from 2 to $T \geq 3$, or ∞ .
 - **Policy**: observe the behaviour of functions if the inflation changes exogenously after the rate cut. Observe also the impact of “helicopter-money” styled consumption boost, then to interpret the contemporary Japanese macroeconomic phenomena.
 - **Expectation**: set a proper expectation for the futures, e.g. stochastic expectation on prices with uncertainty.
 - **General Equilibrium**: generalise the price of banana and land to be endogenous, meanwhile ensure the system is solvable. Generalise the number of goods to $N \geq 2$.
 - Adjusting towards a **DSGE structure**, e.g. make changes in probability distributions, broader the General Equilibrium to include international trade.
- Extension.
 - **Microeconomic Extensions**: information disparities, degree of competitions amongst commercial banks, and risk-taking financial institutions.
 - **Econometric Verifications**: Japanese commercial banks' balance sheet data, and macroeconomic data compared to the model.

Conclusion at the current stage and Evaluation

- Conclusion at current stage.
 - Without any further assumption, answer to the topic question is **uncertain**.
 - ▶ If I assume (restrict) the coefficients and functions to be such that the commercial bank faces a substantial drop in profit after the rate cut,
 - ▶ then the result can be proved as a **clear NO**, i.e. negative interest rate does not boost consumption and investment in Japan.
- Evaluation: Robustness and Deficiency.
 - Potential critique: use of functional and topological analyses overkill the problem. **But rigorous mathematical proofs ensure robustness.**
 - Reliance on some microeconomic assumptions, e.g. separable utility functions over time. **(The same problem amongst some current literature.)**
 - Until full conversion to a DSGE structure, this paper is not on a publishable level, thus may not be a strong policy recommendation. **(Vice versa.)**
- Evaluation: Strength and Summary.
 - **High quality analyses** demonstrating excellent knowledge on relevant literature, accurate and rigorous mathematical foundation.
 - **Original work** which brings new concepts to the dynamic macroeconomic models, e.g. using different interest rates with some preserving Zero Lower Bound.
 - Provides **theoretical basement** for further empirical works.